Physics

Unit 5 –
Unbalanced Forces & Newton's 2nd Law

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Unit 5: Unbalanced Force Models (Newton's 2nd Law of Motion)

Newton's Laws of Motion:

1.Objects in motion will remain in motion with constant velocity unless acted upon by unblanced forces. (This means that if an object is not accelerating, then the forces acting on it must be balanced.)

2.$\Sigma F=ma$. (Net Force = Mass x Acceleration)

3.For every action force there is an equal and opposite reaction force.

Newton's 2nd Law

Newton's Second Law is the main focus of Unit 5. In the lab we determined that when the amount of force is increased, the acceleration also increases. We found that as the mass increased, the acceleration decreased. This relationship was quantitatively determined to be F = ma, where the force was actually the NET FORCE acting on the system.

Finding the net force occurs in one of two ways—First, as the equation states, you could find the NET FORCE by taking the objects **mass** time **acceleration**. Second, you could find the NET FORCE by analyzing the individual forces acting on an object. To do this,

•Begin with a Force Diagram.

•Identify the direction the system will accelerate.

•Line up one of your coordinate axes with the acceleration.

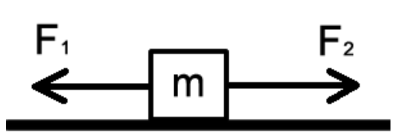
•Decide a direction for positive.

•Add up the forces (or force components) that lie in the positive direction.

•Subtract the forces (or force components) that lie in the negative direction.

Finding the NET FORCE by looking at forces:

Suppose there is a “tug of war” in which one team pulls to the right with 500 N of force, and the other team pulls to the left with 450 N of force. If the diagram represents this situation, F_1=450N and F_2=500N. So the $\Sigma F = F_2 - F_1$. The *NET FORCE* would be 50 N to the right (taking *right* to be positive, we get 500 N – 450 N = 50 N).



Suppose $F_1 = 95$ N and $F_2 = 75$ N. What would the Net Force be?

•Since there is more force to the left, we'll go ahead and call LEFT the positive direction.

•This means that $\Sigma F = F_1 - F_2$.

•So **ΣF** = 95 – 75 = **20 N** to the ***LEFT***.

Finding the NET FORCE by taking Mass x Acceleration

We know that Newton's 2nd Law states that the Net Force = Mass x Acceleration, so if we know the mass and the acceleration of an object, we can find the Net Force by simply multiplying the two numbers together.

For example, suppose a 50 kg object is acclerating at 2.5 m/s^2. What Net Force must be acting on the object?

•$\Sigma F = ma$

•$\Sigma F = (50\text{ kg})(2.5\text{ m/s}^2)$

•$\Sigma F = 125$ N

Newton's 2nd Law and Kinematics

You have noticed that the formula for Newton's 2nd Law ($\Sigma F=ma$) includes *acceleration*. We have dealt with acceleration before—Unit 3 to be specific! We learned how to find various descriptors of motion for an object using the Kinematics Equations. Below are the equations. Remember, there are five kinematics variables—initial velocity (v_o), final velocity (v), acceleration (a), displacement (d), and time (t). Once you know any three of these, you can solve for either of the other two.

$d = vt$	**constant velocity only**
$d = 1/2\ (v+v_o)t$	don't need a
$v = v_o + at$	don't need d
$d = v_o t + 1/2\ at^2$	don't need v
$v^2 = v_o^2 + 2ad$	don't need t

Solving Unbalanced Force Problems

The steps involved in solving unbalanced force problems are very similar to those for solving balanced force problems. Let's review:

Review:

Solving *Balanced* Force Problems:

1.Draw a force diagram.

2.Write "Up=Down" and "Left=Right"

3.Identify the Up, Down, Left, and Right components of forces.

4.Plug in known values, draw triangles to find missing components, solve.

New:

Solving *Unbalanced* Force Problems:

1.Draw a force diagram. Identify the direction it will accelerate. Line up one of your axes with the acceleration.

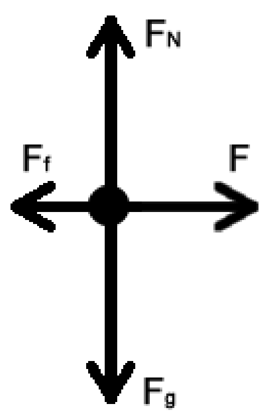
2.Write **ΣF=ma** *(for the direction it accelerates)*

3.Identify the Up, Down, Left, and Right components of forces.

4.Plug in known values, draw triangles to find missing components, solve. *(This may involve using kinematics equations.)*

Example 1: *You push a 20kg box to the right with a force of 50N. If 15N of friction are present, what is the acceleration of the box?*

Draw the force diagram.



Because there is more force to the right than there is to the left, it will accelerate to the right. So we make right the positive direction.

$$\Sigma F = ma$$

$$F - F_f = ma$$

$$50 - 15 = 20 \cdot a$$

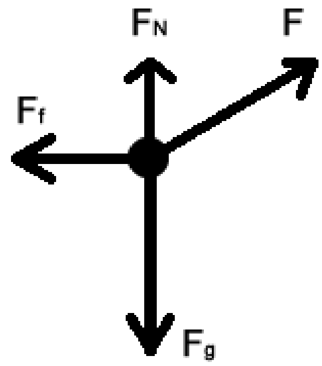
$$35 = 20 \cdot a$$

$$35/20 = a$$

$$\mathbf{1.75\ m/s^2 = a}$$

Example 2: *You attach a rope to a 20kg box, and pull it to the right with a force of 50N at an upward angle of 25°. If 15N of friction are present, what is the acceleration of the box?*

Draw the force diagram.



Because there is more force to the right than there is to the left, it will accelerate to the right. So we make right the positive direction. Note that it accelerates to the right, so we only need to consider forces (components) that are right or left. The applied force F is at an angle. We only use the part of it that is to the right—we call the horizontal component of the force $\mathbf{F_x}$. To find $\mathbf{F_x}$ we need to draw a triangle, label it, and then use trig (sine, cosine, or tangent) to find it.

$$\mathbf{\Sigma F=ma}$$

$$\mathbf{F_x - F_f = ma}$$

$$(F=50N,\ \theta=25^o)$$

$$(\cos 25^o = F_x / 50)$$

$$(50\cdot\cos 25^o = F_{x)}$$

$$(45.3N = F_{x)}$$

$$\mathbf{F_x - F_f = ma}$$

$$\mathbf{45.3 - 15 = 20\cdot a}$$

$$\mathbf{30.3 = 20\cdot a}$$

$$\mathbf{30.3 / 20 = a}$$

$$\mathbf{1.52\ m/s^2 = a}$$

Example 3: *How far will the box in Example 2 travel in 8.0 seconds if it began at rest?*

For this problem we will have to use kinematics, after we used **ΣF=ma** to find the acceleration.

$$\mathbf{v_o = 0\ m/s}$$

$$v = \text{(don't need v)}$$

$$\mathbf{a = 1.52\ m/s^2}$$

$$\mathbf{d = ???}$$

$$\mathbf{t = 8.0\ s}$$

$$\mathbf{d = v_o t + \tfrac{1}{2} a t^2}$$

$$d = 0 \cdot 8 + \tfrac{1}{2} \cdot (1.52)(8.0^2)$$

$$\mathbf{d = 97.3\ m}$$

Friction

When we first learned about the force of friction, we learned that it is a contact force between an object and a surface which is always parallel to the surface, and resists motion. This is an accurate definition. Remember, though, that there is another force we learned about that results from the same interaction—contact between an object and a surface—the Normal Force. Because the two forces result from the same interaction, they are very closely related. As a result there is a fairly simple formula that relates Friction and Normal forces. Friction is equal to a *number* (known as the ***coefficient of friction***, or μ) times the Normal Force.

$$F_f = \mu \cdot F_N$$

The coefficient of friction is a number, usually less than one, that has no units. For example, if a certain situation had a Normal force of 50 N, and μ was 0.4, then to find the force of friction, you would simply plug into the formula and multiply: $F_f = (0.4)\cdot(50) =$ **20 N**.

You can rearrange this formula to solve for μ as well.

$$\mu = F_f / F_N$$

Static Friction vs. Kinetic Friction

There are actually two different kinds of friction—static friction and kinetic friction. **Static friction** occurs when the surface and the object are at rest, relative to each other. There is no motion between the surfaces. **Kinetic friction** occurs when the surface and object move relative to each other—think of it as the object *slides* along the surface. Kinetic friction is always less than the maximum possible static friction for any given situation.

Because there are two different kinds of friction, there are two different *coefficients of friction* too. The coefficient of kinetic friction (μ_k) is used for kinetic friction, and the coefficient of static friction (μ_s) is used for static friciton.

Static friction is an interesting kind of force, because it will basically *match* an opposing force up to its maximum value of $\mu_s \cdot F_N$. (Kinetic friction is always equal to $\mu_k \cdot F_N$.) The following example will help you understand this:

Example 4*: A 6 kg block is at rest on a level table with* $\mu_s = 0.50$. *If you push on it to the right with 10N of force, how much friction is present?*

The Normal Force will equal the Force of Gravity in this example. ($F_g = mg = (6)(10) = 60N$)

The maximum possible static friction, then, is $\mu_s \cdot F_N = (0.50)(60) = 30$ N.

So if you push with 10 N to the right, will there really be 30 N of friction pushing left? No way. It will match your 10 N push, so the static friction will be 10 N to the left. In order to make the block move, you need to overcome static friction. Just how hard would you have to push it to get it moving? Well, it needs to be only a negligible amount above $\mu_s \cdot F_N$, which in this case means 30 N. Once you push with 30 N, you overcome static friction, and the block begins to move. Then kinetic friction takes over. If you continue to push with 30 N, the block will accelerate.

Example 5: *A 6 kg block at rest begins sliding on a level table with* $\mu_k = 0.30$ when *you push on it to the right with 40N of force, how fast will it be moving after 4.0 seconds?*

We first will need to use **ΣF=ma** to find the ***acceleration***.

Then we will use kinematics to find the displacement.

$$\Sigma F = ma$$

$$F - F_f = ma$$

$$F_f = \mu_k \cdot F_N = (0.30)(60) = 18 \text{ N}$$

$$40 - 18 = 6 \cdot a$$

$$22 = 6 \cdot a$$

$$22 / 6 = a$$

$$3.67 \text{ m/s}^2 = a$$

Now for the kinematics part...

$$v_o = 0$$

$$v = ???$$

$$a = 3.67 \text{ m/s}^2$$

d = ***(don't need d)***

$$t = 4.0 \text{ seconds}$$

$$v = v_o + at$$

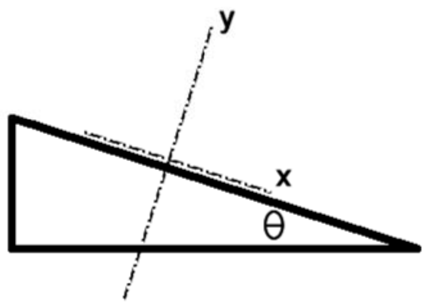
$$v = 0 + (3.67)(4.0)$$

$$v = 14.7 \text{ m/s}$$

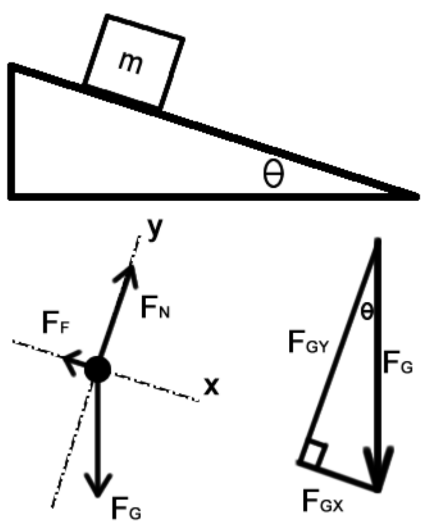
Ramp Problems:

We often encounter objects that are on ramps or inclines. When this occurs, the only thing we need to remember is that the object *would* accelerate along the ramp. Let's choose to make the surface of the ramp our x-axis. Then perpendicular to the surface would be our y-axis. (This is especially convenient since friction is parallel to the surface and normal is perpendicular to the surface.)

Ramp problems often involve finding x and y components of the force of gravity.
(**F_{GX}** and **F_{GY}**) When drawing the triangle for this force, remember to begin with the **F_G**, then draw the legs of the triangle so they line up with your x and y axes. *The angle between **F_G** and the y-axis will be the same as the ramp's angle measured from the horizontal.*



In the example below, I chose downhill to be the positive X direction (because that is the direction the block would accelerate if it does!).



So, to find **F_{GX}** you would use the ***sine*** of θ, and to find **F_{GY}** you would use the ***cosine*** of θ.

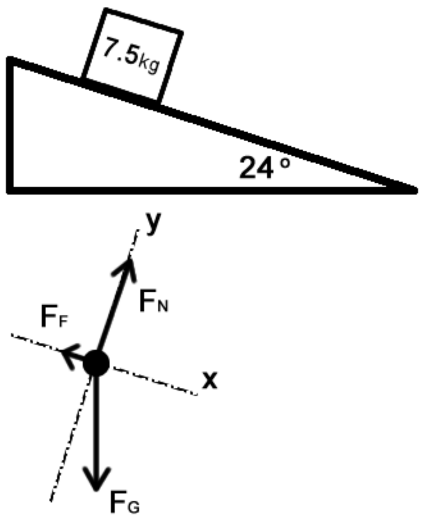
Once you have figured out how to find **F_{GX}** and **F_{GY}** you would be able to use U=D and L=R with balanced force problems.

Because F_g=mg, we can always find F_{gx} and F_{gy} on a ramp problem:

$$F_{gx}=mg\cdot\sin\theta$$

$$F_{gy}=mg\cdot\cos\theta$$

Example 5: *A 7.5 kg block is released from rest on a 24º ramp that has 16 N of friction. Find the Acceleration. Then find how long it will take to slide 1.2 m down the ramp.*



$$\Sigma F = ma$$

$$F_{gx} - F_f = ma$$

$$F_{gx} = mg\sin\theta = (7.5)(10)(\sin 24°) = \mathbf{30.5\ N}$$

$$F_{gx} - F_f = ma$$

$$30.5 - 16 = 7.5\cdot a$$

$$14.5 = 7.5\cdot a$$

$$14.5 / 7.5 = a$$

$$\mathbf{1.93\ m/s^2 = a}$$

Now we will use this Acceleration with a kinematics equation to find how long it will take to slide 1.2 meters.

$v_o = 0$

v = (don't need v)

$a = 1.93\ m/s^2$

$d = 1.2$ m

$t = ???$

$d = v_0t + \frac{1}{2} at^2$

$1.2 = 0 \cdot t + (\frac{1}{2})(1.93)(t^2)$

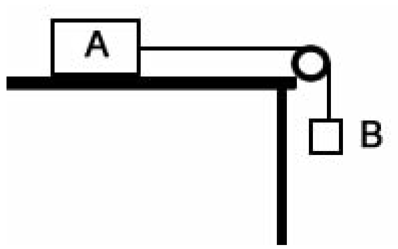
$1.2 = 0.965 \ t^2$

$1.24 = t^2$

square root both sides to get 't' by itself.

1.11 s $= t$

Multiple Object Systems

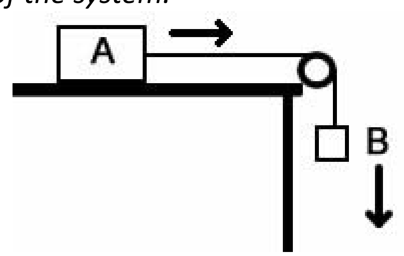


In the lab we performed, we had a cart on a horizontal track, with a string looped over a pulley and a weight hanging off the end, as shown above. Problems of this nature usually involve finding the acceleration and maybe the tension in the string. There are actually a couple of different ways we could approach a multi-object system.

Single System Approach:

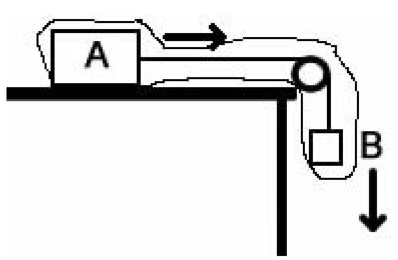
The first method we will discuss is what I call the single system approach. In this approach, we treat each object of the system collectively as a single system. Identify the direction each object accelerates and call that positive. Then look only for forces that either add to or take away from the acceleration. Apply these forces to $\Sigma F = ma$, and be sure to use the entire mass of the system as the ***m***.

Example 6a: *A 5 kg block is on a horizontal table (μ=0.05). A light string (meaning we can neglect its mass) attached to the cart is placed over a frictionless pulley, and a 0.7 kg mass is attached to the other end. Find the Acceleration of the system.*

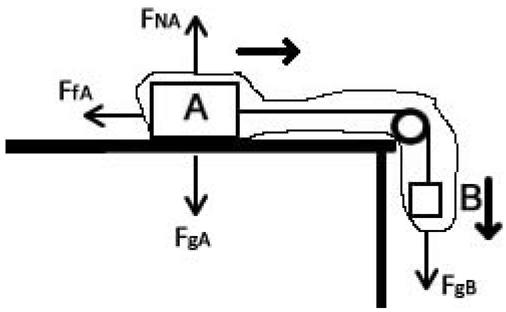


Identify the direction each part of the system accelerates.

Draw a line around the system. Note that to the ***right*** *for block A is the same direction as* ***down*** *is for block B.*



Identify the EXTERNAL forces acting ON the system... note that Tension is an INTERNAL force when we choose a single system approach.



In this example, F_{gA} and F_{NA} do not affect the acceleration, so the Net Force is just $F_{gb}–F_{fA}$.

$$\Sigma F=ma$$

$$F_{gb}-F_{fA}=(m_A+m_B)a$$

$$7-(0.05)(50)=5.7a$$

$$4.5=5.7a$$

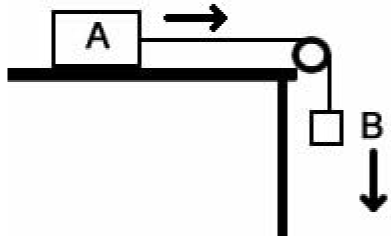
$$0.79m/s^2 = a$$

Two System Approach:

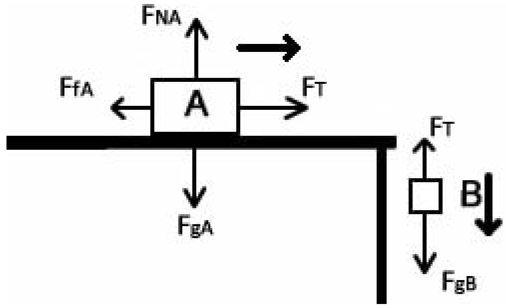
The second method we will discuss is what I call the two system approach. In this approach, we treat each object of the system individually, so there are two different systems you need to work with. Identify the direction each object accelerates and call that positive. Follow the steps of solving force problems for each object separately...eventually you reach a point where there are two unknown values —Tension and Acceleration.

Solving two equations with two unknowns is sometimes referred to as a System of Equations in a math class. While there are a variety of methods of solving systems of equations, I will teach you just one—Substitution/Elimination. In this method, you solve one of the equations for one of the variables, then substitute that into the other equation. For example, solve one equation for the Tension, then plug in what you get into the other equation where you see Tension.

Example 6b: *A 5 kg block is on a horizontal frictionless table. A light string (meaning we can neglect its mass) attached to the cart is placed over a frictionless pulley, and a 0.7 kg mass is attached to the other end. (μ=0.05) Find the Acceleration of the system.*



Identify the direction each part of the system accelerates. This will be the positive direction.



Draw a force diagram for each separate object in the system. Remember the direction each one accelerates is the positive direction.

Now do $\boldsymbol{\Sigma F=ma}$ *for each object separately. Here is what that looks like--*

Object A:

$$\Sigma F=ma$$

$$F_T-F_{fA}=m_Aa$$

$$F_T-(0.05)(50)=5a$$

$$\underline{F_T-2.5=5a}$$

Now solve this one for Tension: $F_T=5a+2.5$

Object B:

$$\Sigma F=ma$$

$$F_{gB}-F_T=m_Ba$$

$$7-F_T=0.7a$$

Now plug in the Tension from Object A:

$$7-(5a+2.5)=0.7a$$

$$7-5a-2.5=0.7a$$

$$4.5=5.7a$$

$$0.79\ m/s^2=a$$

Notice that we got the exact same answer using the two system approach that we got during the single system approach.

Finding the Tension:

Whether you used a single system approach or a two system approach to find the acceleration, in order to find the Tension in a two body system, you need to split the system, and look at a part of the system which does have a tension force. Take object B for example. Now that you know the acceleration, finding the tension is fairly easy.

Object B:

$$\Sigma F=ma$$

$$F_{gB}-F_T=m_B a$$

$$7-F_T=(0.7)(0.79)$$

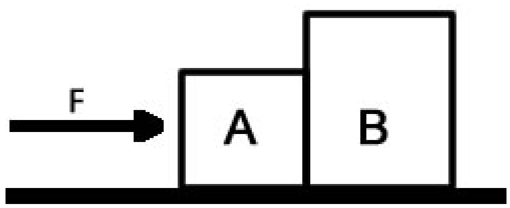
$$7-F_T=0.553$$

$$7=0.553+F_T$$

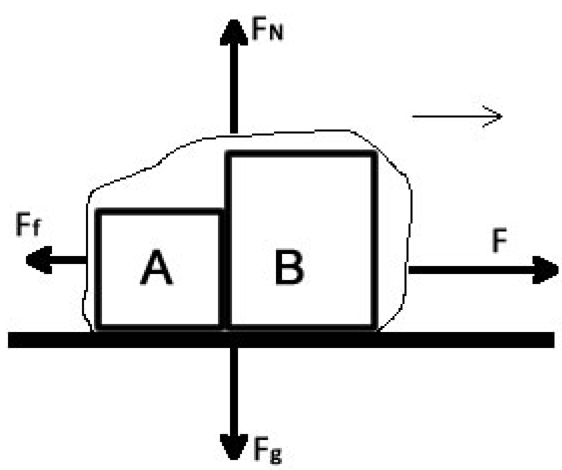
$$6.45N=F_T$$

* The same approach can be applied to blocks *pushing* against each other as well. Use either a single system approach or a two-system approach to find the acceleration, then split them up to find the contact (pushing) force between them.

Example 7a: *A student pushes on Block A (5 kg) to the right with a 75 N force. Block B has a mass of 12 kg. (μ=0.2) Find the Acceleration of the system.*



Single system--



It accelerates to the right, so our Net Force will be Right minus Left. (Note the Normal Force here equals the combined weight of the two blocks.)

$$\Sigma F=ma$$

$$F-F_f=(m_A+m_B)a$$

$$75-(0.2)(170)=17a$$

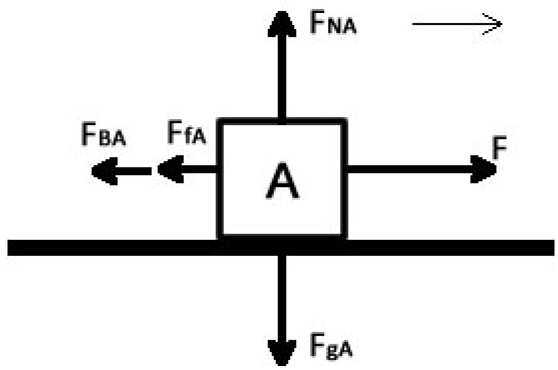
$75-34=17a$

$41=17a$

$2.41\ m/s^2 = a$

Two-system--

There is a contact force between Blocks A & B. We'll call the contact force that A pushes ***on*** *B* ***F_{AB}*** *and the contact force that B pushes* ***on*** *A* ***F_{BA}. Notice that this is an action-reaction pair, so they are equal in magnitude and opposite in direction. In the equations, we deal with the direction by choosing the acceleration to be the positive direction, so the important thing to recognize here is that*** *$F_{AB}=F_{BA}$.*



$\Sigma F=ma$

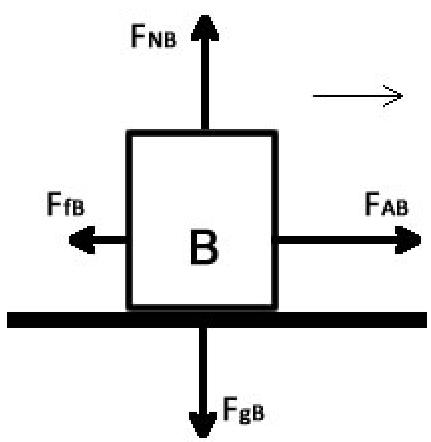
$F-F_{fA}-F_{BA}=m_A a$

$75-(0.2)(50)-F_{BA}=5a$

$65-F_{BA}=5a$

$65=F_{BA}+5a$

$\underline{65-5a=F_{BA}}$



$\Sigma F=ma$

F_{AB}-F_{fB}=m_Ba

F_{AB}-(0.2)(120)=12a

<u>F_{AB}-24=12a</u>

Now plug in the Contact Force (F_{AB}) from block A:

(65-5a)-24=12a

65-24=17a

41=17a

2.41 m/s^2 = a

We can find the contact force by plugging this accleration into either equation of the two-system approach:

65-5a=F_{BA}

65-(5)(2.41)=F_{BA}

52.95 N = F_{BA}

or

<u>F_{AB}-24=12a</u>

F_{AB}-24=(12)(2.41)

F_{AB}-24=28.92

F_{AB}=52.92 N

(The very slight (negligible) difference is because of rounding the acceleration.)