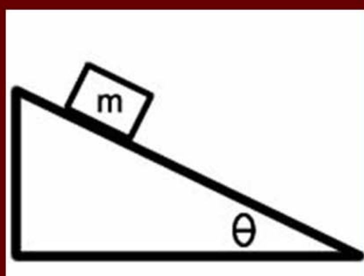
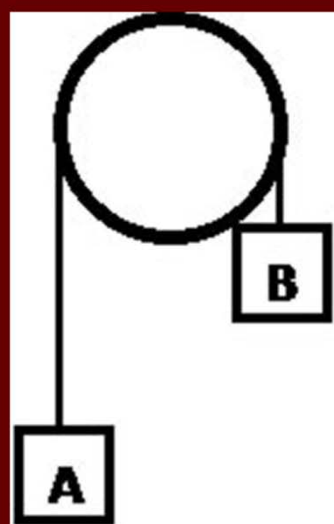
PHYSICS

UNIT 7: WORK, ENERGY, POWER





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Physics-Unit 7: Work, Energy, & Power

By Tom Erekson

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Spring Force

Consider what happens to a spring when you apply a force to it. It will stretch. The farther you stretch it, the greater the amount of force with which the spring pulls back. When the relationship of Spring Force vs. Stretch is graphed, we see a linear graph. The slope of this graph is what we call the Spring Constant. The symbol k is used to denote the spring constant. The equation of the linear graph is $F_{spr}=kx$. This relationship is known as Hooke's Law.

Work

Constant Force, Same Direction

In a junior high physical science class you may have learned about the concept of work. In that setting work was defined by the formula: Work = Force x Distance. This equation is accurate, but only under specific conditions. Taking Force times Distance will only give us the Work if the Force is constant and in the same direction as the Distance. So if a 20 N force is pushing to the right on a box that moves 3 m to the right, then we can see that the work done would be (20 N)(3 m) = 60 Nm. This basic formula is accurate because the force was constant, and it was in the same direction as the distance.

Constant Force, Different Direction

What happens when we have a constant force, but the force is not in the same direction as the displacement? In such a case we need to find the component of the force that is in the direction of the displacement, and then we can multiply those two quantities together. The parallel component of a force can be found by taking the cosine multiplied by the force. $F_x = F \cos\theta$. So Work = $(F \cos\theta)(d)$, or as it is more commonly written, $W=Fd \cos\theta$. This formula works for any constant force. The angle in the formula is the angle between the Force vector and the Displacement vector. (Recall that a vector is a quantity that has both magnitude and direction.)

Non-Constant Force

So now we know how to calculate the work done by a constant force, even when it doesn't line up with the displacement. But how do we find the work done by a non-constant force? Let's consider the Spring Force as an example of a non-constant force. We know that the farther you stretch a spring, the greater the force the spring exerts. In this way, the amount of force actually changes with the amount of stretching. If we were to create a graph of the Spring Force vs. the Stretch (displacement or Δx of the spring), we would see a linear graph. Finding the area under the F vs. X graph will give you the amount of work done. This method works for all forces, whether they are constant or not.

W = Fd (constant force, force in same direction as displacement)

W = Fd cosθ (constant force, θ = angle between F and d)

W = Area under F vs. X graph (any force)

Work is one way that energy can be transferred into or out of a system. The amount of work done is equal to the amount of energy transferred. Let's learn about energy now…

Energy

Energy has been defined as *the ability to do work*, using our mathematical definition of work. This is true, but does not really tell the whole story. For our purposes, we define energy as a conserved, substance-like quantity with the capability to produce change. Energy can be stored in various forms. You have probably heard about several of these forms of energy: Kinetic Energy (Ek), Gravitational Potential Energy (Eg), Elastic Energy (Eel), Dissipated Energy (Ediss), Chemical Energy, Nuclear Energy, Electromagnetic Energy, etc.

An analogy that may help you understand energy storage is to think of energy like money, and energy storage like bank accounts. You can store money in a checking account, savings account, certificate of deposit (CD), money market account, etc. Money can be transferred from one account to another. In much the same way, energy can be transferred from one form to another. The unit we use for energy in the SI (metric system) is the **Joule.** The symbol we use for the Joule is the capital letter J. So transferring 100 J of energy from Kinetic Energy to Gravitational Potential Energy is like transferring $100 from checking to savings. The amount of stuff remains the same, only where it is stored has changed.

Kinetic Energy

Kinetic Energy (Ek) is the energy of motion. Moving objects have kinetic energy. The formula for calculating kinetic energy is:

$$E_k = \frac{1}{2} m v^2$$

In this formula it is important to have units input correctly—**mass** in kg, **velocity** in m/s, **energy** in Joules.

Gravitational Potential Energy

Gravitational Potential Energy (sometimes called simply Gravitational Energy or just Potential Energy) is energy stored in an object by virtue of its position within a gravitational field. That is a fancy way of saying that the higher an object is the more gravitational energy it has (because it would have farther it could fall). The formula for calculating gravitational potential energy is:

$$E_g = mgh$$

Here, **m** is the mass in kg, **g** is the acceleration due to gravity (in m/s^2), and **h** is the height in meters above zero. The convenient thing about gravitational potential energy is that you can pick the location of zero height to be wherever you want it. Often the most convenient place to put zero height is the lowest position in a problem. So if a book is lifted 1.5 m above a table, make the table your zero height (instead of the floor, for example).

Elastic Energy

Elastic energy is the energy stored in elastic substances, like springs or rubber bands. For this unit, we'll treat all elastic substances as if they behaved like springs. The formula for calculating the elastic energy is:

$$E_{el} = \frac{1}{2} kx^2$$

In this formula, **k** is the spring constant in N/m, **x** is the stretch (or displacement from equilibrium) in meters. This formula can be found by taking the area under the F vs. x graph for a spring. We can see that the work done is equal to the energy transferred.

Dissipated Energy

Energy can be dissipated, which is a fancy way of saying that it becomes spread out into the environment. This is usually done through heat, but we can often calculate the amount of dissipated energy by looking at the amount of work done by certain forces, such as friction. The work friction does equals the amount of energy dissipated.

Conservation of Energy

The Law of Conservation of Energy states that energy cannot be created or destroyed, but only change from one form to another. Simply put, this can give us a formula to use to solve problems involving conservation of energy:

Initial Energy = Final Energy

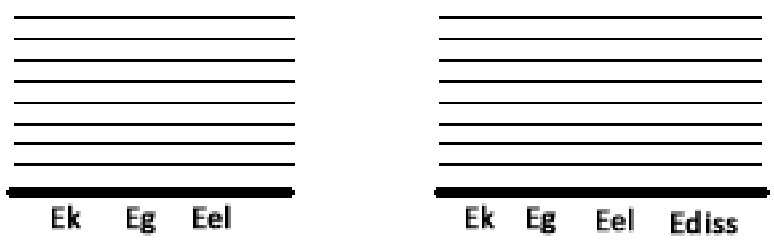
In this unit we are focusing primarily on mechanical types of energy (kinetic, gravitational, elastic, dissipated).

Energy Bar Graphs

We can create energy bar graphs to help us represent the initial and final energies of a given problem. In mechanical types of problems, there are three types of energy that could be present initially: Kinetic, Gravitational, and Elastic. Represent each of these on a bar graph to show the initial energy of a system. We know a system has kinetic energy when it is moving. We know a system has gravitational energy when it is at a height above zero (and we choose zero height to be the lowest point in a problem). We know a system has elastic energy when a spring or other elastic material is either stretched or compressed.

We do the same for the final energy. However, there are a couple of important differences for final energy. Some of the energy may have been dissipated by friction, so a fourth energy type (Dissipated) is available for the final energy. It is also possible that energy has been added to a system by means of work. When that is the case, there is more final energy than there was initial energy. We represent this on an energy bar graph by writing "+W" (meaning positive work) in the space between the initial energy bar graph and the final energy bar graph. This represents an energy flow diagram, where we account for energy that has been added to a system through work. The amount of work done will exactly equal the amount of energy added to the system. (Remember that we account for loss of energy from a system through *dissipated energy*.) So in this way, the bar graph helps us visually see how to set up our conservation of energy equation:

Ei + W = Ef

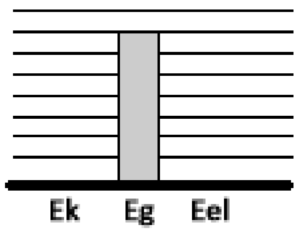


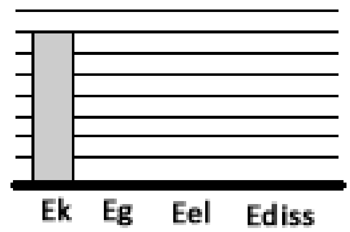
Solving Conservation of Energy Problems

Solving conservation of energy problems is pretty straightforward once you have correctly identified the initial and final energy of a system on an energy bar graph.

Example 1: *A 7 kg box is located 6m above the ground on a hill. If it is released from rest, and there is no friction, how fast will the box be moving at the bottom of the hill?*

Draw the energy bar graph: In this case the box begins with gravitational energy only—it is not moving, and there are no springs, but it is above the lowest position in the problem. The box ends with kinetic energy only—it is now at the lowest point (zero height) so it no longer has any gravitational energy.





Ei + W = Ef

Eg (initial) = Ek (final)

mgh = ½ mv^2

(7)(10)(6) = ½ (7)(v^2)

420 =3.5 v^2

120 = v^2

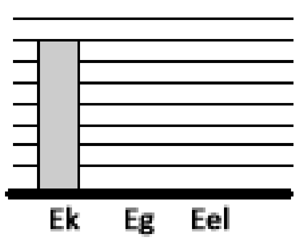
10.954 = v

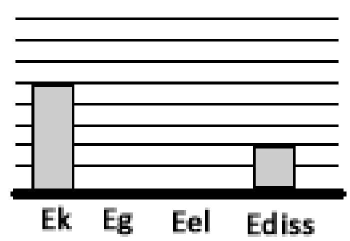
11.0 m/s = v (3 sig figs)

Solving Conservation of Energy Problems Involving Dissipated Energy

When friction is present, there will be dissipated energy. There are a couple of different ways to find dissipated energy. One is to find the work done by friction or other forces.

Example 2: *A 1.2 kg toy car is moving at 3.0 m/s when it hits a stretch of carpet. The coefficient of friction on the carpet is 0.6. How fast will it be moving after traveling 2.5 m across the carpet?*





Ei + W = Ef

Ek (initial) = Ek (final) + Ediss

½ mv_o^2= ½ mv^2 + Wfric

Wfric = Ff*d = μ*mg*d

½ (1.2)(3.0^2) = ½ (1.2)(v^2) + (0.6)(1.2)(10)(2.5)

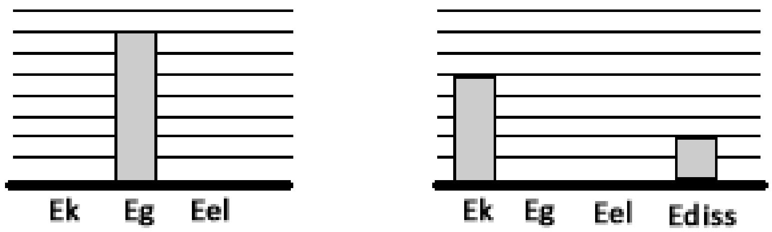
5.4 = 0.6 v^2 + 1.8

3.6 = 0.6 v^2

6 = v^2

2.45 m/s = v

Example 3: *A 1.4 kg rock is dropped from 30 m above a pool of water. It reaches the water with a speed of 20 m/s. How much energy was dissipated by air resistance?*



Ei + W = Ef

Eg (initial) = Ek (final) + Ediss

mgh = ½ mv^2 + Ediss

1.4*10*30 = ½*1.4*20^2 + Ediss

420 =280 + Ediss

140 J = Ediss

Power

Power is defined as the rate at which work is done, or in other words, the rate at which energy is transferred. A *rate* is a measure of something *per unit time*. The formula for Power looks like: Power = Work / time. Now if work = force x displacement, then we could write this as Power = Force x Displacement / time. But we remember that Displacement / time = velocity, so Power could also be calculated using Power = Force x Velocity. A lot of power problems involve changing height in a certain amount of time. So the change in energy would be a change in gravitational potential energy in a certain time. This gives us an equation for power of: P = mgh/t. Any of these equations will give you the same answer for Power if you use them correctly. Look for the one that is easiest to use based on what information is given to you. Power is measured in Joules per second (J/s). This is given the special name of the Watt. One Watt = One Joule per second. So a 100W light bulb consumes 100J of electrical energy per second!

P = W/t

P = ΔE/t

P = mgh/t

P = Fv

Example: Jonny (65 kg) runs up a flight of stairs, gaining 6.3 meters in height. He accomplishes this tremendous feat in 2.7 seconds. How much power did he produce?

P = E/t

P = mgh/t

P = (65)(10)(6.3)/2.7

P = 1520 Watts

(3 sig figs)